DOUGLAS FIR USE BOOK

STRUCTURAL DATA AND DESIGN TABLES

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HYPERBOLIC-PARABOLOID SHELLS

Doubly curved, hyperbolic-paraboloid roof structures of lumber are easy to design and to construct, and have the added benefits of economy and striking appearance. The name stems from the mathematical formulae describing the curvature of the decking surface.

The curve of the decking is a convex parabola when viewed parallel to one axis drawn through opposite corners, and a concave parabola when viewed from an axis 90 degrees to the first. Curves formed by the intersection of the surface of the shell with a horizontal plane are hyperbolic. The horizontal projection of such a shell may be square or diamond in shape, with the four sides of equal length. When a roof structure consists of a number of hyperbolic-paraboloids, the horizontal projection of an individual shell may be rectangular in shape with adjacent sides of unequal length. In elevation, the opposite corners are elevated an equal distance above the other two. (See Figure 84).

The principal forces to be considered in the design of a hyperbolic-paraboloid shell are the reactions, the compression forces in the perimeter members, the shear forces at the junction of the sheathing and the perimeter members, and the direct tensile and compression forces in the sheathing. These forces are easily determined by statics. Once the forces have been determined, the member sizes and connections are designed by standard engineering procedures. The method of analysis described here can also be applied to a series of hyperbolic-paraboloids.

While there are several methods of computing the forces in a hyperbolic-paraboloid shell, the simplest is to resolve the reactions into component forces in the surface and the perimeter members. This method of analysis can best be illustrated by the following example:

Figure 85 is an isometric view of a simple shell where:

- a = length of one side,
- a' = length of the horizontal projection of length a,
- C = total compression force in perimeter member,
- c = principal compressive force in sheathing per foot,
- F = resultant of the vertical reaction R and the horizontal thrust H,
- H = horizontal thrust,
- h = vertical distance from a support to the highest point of the shell,
- k = inclined distance from a support to the mid-point of the length l,
- l_1 = length along longitudinal axis,
- l_2 = length along transverse axis,
- R = vertical reaction,
- t = principal tension force in sheathing per foot,
- v = boundary shear force per foot.

Loading Conditions:

Because of curvature, the dead load is not uniform over the projected area and the variance increases as the rise increases. However, for hyperbolic-paraboloid shells built of lumber the dead load is likely to be very small as compared to the live load, and the non-uniform distribution of the dead load is therefore usually neglected.

Unsymmetrical concentrated loads or un-

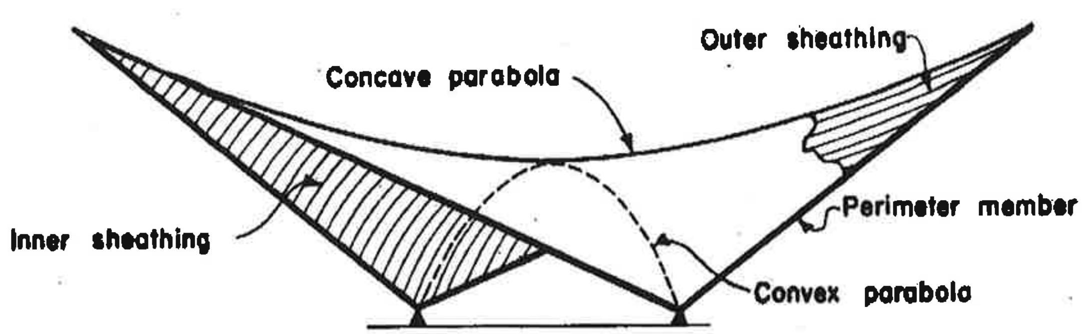

Figure 84—Single Hyperbolic-Paraboloid Shell.

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balanced live loads could cause the structure to tip unless restrained by:

1. A tension tie or strut from foundation to each high point. (Wall framing may be designed to serve this purpose.)
2. A combination tension and compression tie to one high point.
3. Connections at the two principal points of support especially designed to resist rotation.
4. Multiple hyperbolic-paraboloid structures, inter-connected at their adjacent high points.

Restraint of the high points that prevent normal deflections of the shell under live load will cause secondary stresses in the structure. However, the load, and therefore the deflections, at the high points are very small. It is suggested that these secondary stresses be neglected, since the unit stresses in the shell are normally very low.

Since wind, snow or other live loads are normally unbalanced as they occur on structures, the restraint against tipping must be provided even though the design assumes balanced loading. Because the structure's dead weight is small, it should be well tied down during erection to resist wind forces.

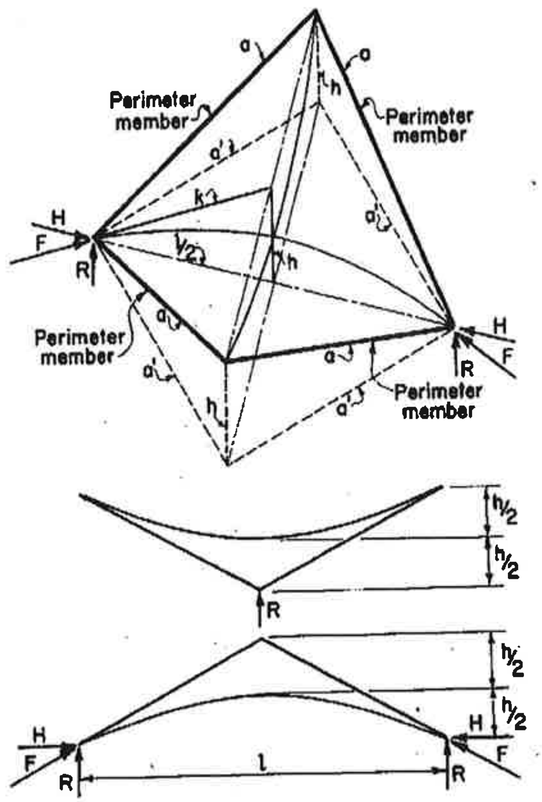

Figure 85—Geometry of Hyperbolic-Paraboloid.

Force in Perimeter Members:

For balanced live and dead loads the vertical reactions are one-half the sum of the vertical loads. The horizontal thrust, H, at the reactions is derived by simple proportion, and for this condition (see Figure 85), the thrust is determined as follows:

$$\frac{R}{h} = \frac{H}{l/2}$$

solving for H;

$$H = \frac{Rl}{2h} \qquad (1)$$

The force, "F," which is in the direction of the line k in Figure 85, is the resultant of the horizontal thrust, "H," and the vertical reaction, "R."

"F" is solved by proportion as follows:

$$\frac{F}{k} = \frac{R}{h}$$

solving for F;

$$F = \frac{Rk}{h} \qquad (2)$$

Dividing the force, "F," into components parallel to the perimeter members, the compression force, "C," in the perimeter members is:

$$\frac{F/2}{k} = \frac{C}{a}$$

solving for C;

$$C = \frac{aF/2}{k}$$

substituting the value for F in equation (2)

$$C = \frac{a\frac{Rk}{h}}{2k} = \frac{Ra}{2h} \qquad (3)$$

The foregoing is based on the assumption that the reactions will be at the two low points of the structure. If the supports are placed at the two high points the forces will

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be the same, except that the perimeter members will be in tension instead of compression and the horizontal thrusts will be in the opposite direction.

Boundary Shear:

There is a shearing force along the length of the perimeter members at the junction of the sheathing and the perimeter members. This boundary shear, "v," in pounds per lineal foot, is found by dividing the compressive force, "C," in the perimeter member by the length of the perimeter member.

Arrangement of Lumber Sheathing:

Lumber sheathing is ideal for shells because the comparatively narrow widths of the boards permit easy adjustment to the doubly curved surface without special cutting and fitting except for end trimming to length.

Lumber sheathing may be applied to hyperbolic-paraboloid shells in two ways:

1. One layer of sheathing boards placed parallel to the transverse axis where each piece bows to fit the curve of the convex parabola with a second layer placed parallel to the longitudinal axis where pieces bow to fit the curve of the concave parabola.
2. One layer of sheathing boards placed parallel to two opposite sides of the structure and a second layer placed parallel to the other two sides. In this system, each sheathing board twists slightly. The total amount of twist from perimeter member to perimeter member depends on the slope of the perimeter members.

If the horizontally projected shape of the shell is a square, the layers of roof sheathing are at right angles to each other for both systems of placement. If the shape of the structure is that of a diamond, the double layer of sheathing boards will be at right angles to each other when placement follows the first system. When placed by the second system, the angle between layers of boards will depend on the angles between perimeter members.

As there is a slope, but no curvature, to a series of straight lines from one edge to an opposite edge and parallel to a side, falsework placed in this manner will automatically generate the doubly curved surface and serve as support for the placing of the sheathing.

The first system of placing sheathing boards is advantageous because the principal tension and compression forces in the sheathing act in the direction parallel to the grain, which provides the most efficient use of the sheathing.

For small shells, the second system can be used to advantage as the sheathing boards do not curve and have only a slight twist from end to end. If the shell is small enough so that the sheathing is without undesirable deflection, the falsework can be omitted. This system has one disadvantage since the joints between constant width sheathing boards would leave gaps varying in width from a maximum at the ends to no gap at the midpoint. If this is undesirable, the sheathing boards may be tapered to fit snugly.

Stress in Sheathing:

Since the stresses in the sheathing result in boundary shears along the perimeters, the boundary shears can, conversely, be resolved to determine the stresses in the sheathing. The principal forces in the shell are tensile forces, "t," parallel to the direction of the concave parabolas; and compressive forces, "c," parallel to the direction of the convex parabolas (see Figure 86). When the horizontal projection is a diamond shape, the principal tension and compression forces parallel to the longitudinal and transverse axes can be resolved by proportion as follows:

Referring to Figure 86, the principal tensile force per foot of width, "t," is:

$$\frac{t}{v} = \frac{l_1/2}{a'} \text{ and } t = \frac{l_1 v}{2a'}$$

The principal compressive force per foot of width, "c," is:

$$\frac{c}{v} = \frac{l_2/2}{a'} \text{ and } c = \frac{l_2 v}{2a'}$$

When the horizontal projection of a hyperbolic-paraboloid is square in shape, the principal tension and compression forces per foot of width are equal in magnitude to the boundary shear forces per foot of length of perimeter members.

The unit tensile stress in the sheathing lumber is equal to the principal tension force, "t," per inch of width divided by the thickness of the sheathing in inches that parallels the longitudinal axis. The unit compressive stress in the sheathing which acts at 90 degrees to the tensile stress is equal to the principal compressive force, "c," per inch of width divided by the thickness of the sheathing in inches paralleling the transverse axis.

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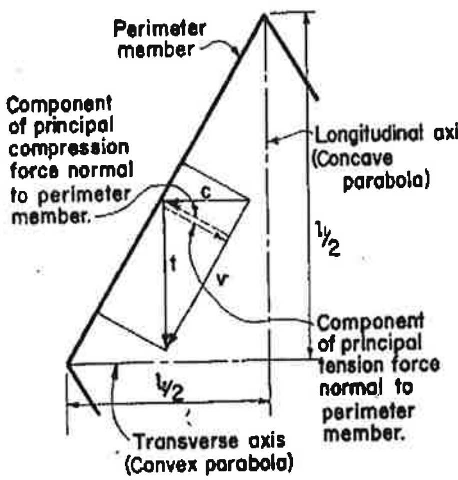

Figure 86—Stresses in Sheathing.

The component of the compressive stress normal to the perimeter member exerts an outward thrust on the perimeter member and the component of the tensile stress normal to the perimeter member exerts an inward pull on the perimeter members. These components, being equal and opposite in direction, as can be seen from Figure 86, will cancel each other, with the result that the perimeter members are subjected only to axial compression forces.

Light concentrated loads produce localized membrane stresses and tend to cause local buckling in the vicinity of the load. These effects can be ignored in design since they dissipate rapidly a short distance away from the load. Also the unit stresses in these shells from the usual loadings are quite low.

Sheathing Parallel to Diagonals:

With the sheathing placed in the directions parallel to the longitudinal and transverse axes of the hyperbolic-paraboloid, each layer acts independently; the layer parallel to the longitudinal axis transmitting the principal tensile forces to the perimeter members and the layer parallel to the transverse axis transmitting the principal compression forces to the perimeter members. Nailing or stapling the layers together, although not required for strength, is required to prevent buckling of the compression layer and will impart additional stiffness to the shell by providing interaction between the layers. Where butt joints occur in the sheathing, additional fastenings should be used to transfer the forces across the joint.

Another method of increasing the stiffness of the shell is to nail-glue the layers of sheathing together in a zone around the perimeter of the shell. The width of the zone varies depending on the size of the hyperbolic-paraboloid, but is generally about one-tenth of the length of a side. This method should be used with extreme caution as there are many uncertainties connected with field gluing.

Sheathing Parallel to Sides:

When the sheathing layers are placed with the boards parallel to the sides of the structure, each layer of boards is at an angle to the direction of the principal tension and compression forces with each layer resisting a portion of the principal tension force and a portion of the principal compression force. As the joints between adjacent boards in a layer represent a discontinuity, these forces have to be transferred across the joints through the adjacent layer by means of the fastenings connecting the layers. This results in a shear between the two sheathing layers which must be resisted by the fastenings. The shear force is equal in magnitude to the boundary shear stress per unit of length which can be converted to a shear stress per unit of surface area.

Design Considerations:

In the design of the component members of a hyperbolic-paraboloid structure, the sheathing resists the principal tension and compression forces parallel to the longitudinal and transverse axes. As the unit stresses in the sheathing are generally quite low, economical, one-inch nominal thickness lumber can be used for the sheathing material.

Where the underside of the sheathing is left exposed to serve as the finish ceiling, the nails or staples should be of a length that will not penetrate completely through the bottom layer of sheathing. They should be of large enough diameter, however, to develop the required lateral and withdrawal resistance.

The boundary shear in the sheathing is transferred to the perimeter members by means of the fasteners used to connect the sheathing to the perimeter members. In this respect the design of the perimeter connections is the same as for lumber sheathed diaphragms. A detailed discussion of this procedure is given on pages 262 to 274.

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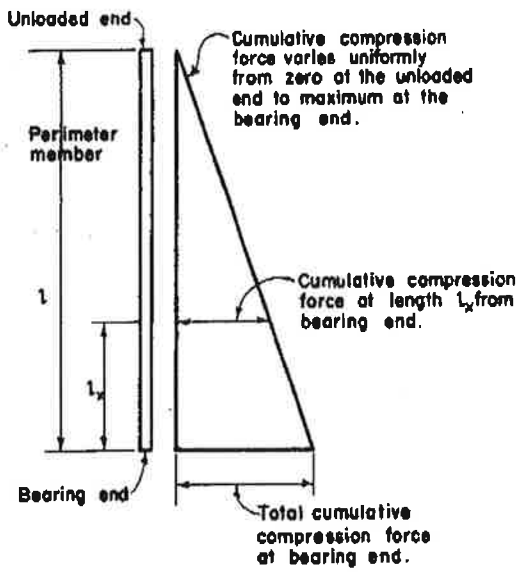

Figure 87—Column Action of Perimeter Member.

The perimeter members transfer all loads to the bearing point and must have sufficient cross section to resist the cumulative axial compressive forces. As the boundary shear forces are distributed uniformly along the length of the perimeter members, the compression force in the perimeter members varies as a cumulative sum of the boundary shear from zero at the high point to maximum at the supports. The perimeter members can be tapered if desired. The sheathing provides lateral restraint to the perimeter members in the direction parallel to the plane of the sheathing. In the direction perpendicular to the plane of the sheathing, the perimeter members receive no lateral support, and the slenderness ratio related to this plane must be considered. As the compressive force varies uniformly from zero at the peak to a maximum at the support, the perimeter members are considered as a series of columns varying from a long column with no load to a short column with maximum load. At any point along the length the induced compression parallel to grain stress due to the accumulated load must not exceed the allowable unit stress as determined by the standard column formula for a column length equal to the distance from the support to the point being considered (see Figure 87). If the sheathing is placed on the top or the bottom of the perimeter members, the boundary shear forces cause bending stresses in the perimeter members due to eccentricity. Hence, the perimeter members are subjected to combined bending and axial compression stresses and must be designed accordingly. If the sheathing is sandwiched into the perimeter members with half of the perimeter members above and half below, there is no eccentricity and the perimeter members are subjected to axial compression stresses only. The latter method permits a somewhat smaller perimeter member to be used, but increases the number of pieces to be framed and handled in erection.

As a hyperbolic-paraboloid shell becomes flatter, it becomes more flexible with an increasing tendency to buckle. For this reason it is desirable to place a limitation on the flatness which can be expressed as the ratio of the rise to the length of a side. A minimum rise-length of side ratio of 1 to 5 is suggested.

Method of Determining Twist in Perimeter Members:

Because a hyperbolic-paraboloid shell is a doubly curved surface, the slope of the sheathing at the junction with the perimeter members is constantly and uniformly changing along the length of the perimeter members. As the surface of the perimeter members must be tangent to the sheathing where they connect, the contacting face of the perimeter members must be shaped appropriately. If the perimeter members are glued laminated members, the changing slope can be obtained by building in a twist to the whole member. For each shell, two right hand and two left hand perimeter members are required.

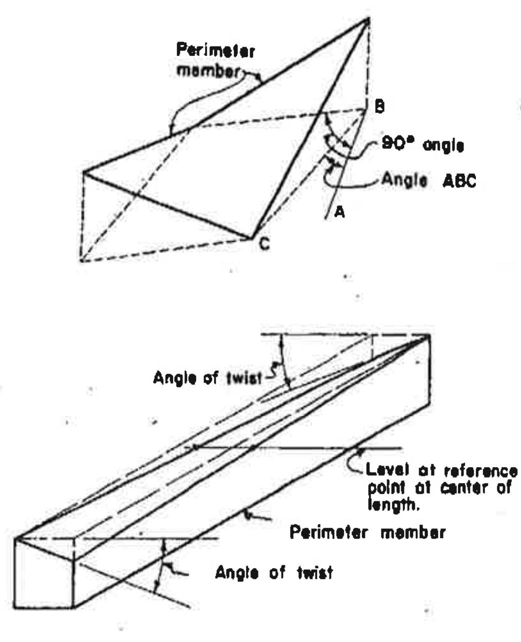

Figure 88—Twist in Perimeter Member.

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The total change in slope, or rotation of the contact surface of the perimeter members from one end to the other, is the angle of twist. As the ratio of the rise in respect to the length of the perimeter member increases, the angle of rotation also increases.

The procedure for computing the angle of twist is applicable to hyperbolic-paraboloids having square or diamond shaped horizontal projections.

Using the mid-point of the length of a perimeter member as a convenient reference, and considering the twist at this point to be zero, the angle of twist from the reference point to either end of the perimeter member as shown in Figure 88, can be determined from the following formulae:

For a hyperbolic-paraboloid having a diamond shaped horizontal projection, the formula for the angle of twist is

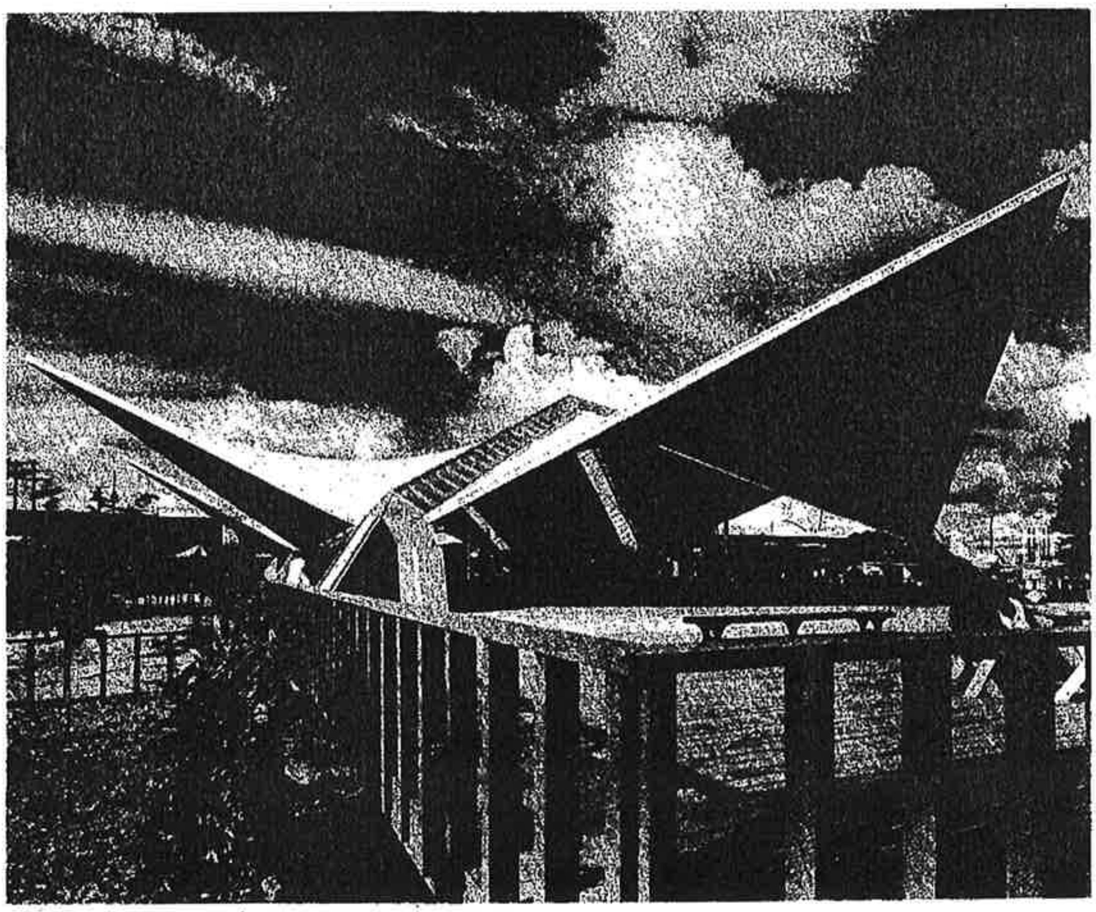
$$\text{tan of angle of twist} = \frac{ha}{(a')^2 \cos ABC}$$

where angle ABC is the angle shown in Figure 88.

When the horizontal projection of the hyperbolic-paraboloid is square, the angle ABC becomes zero and the formula reduces to

$$\text{tan of angle of twist} = \frac{ha}{(a')^2}$$

The total angle of twist from one end of the perimeter member with respect to the other is twice the angle determined from the preceding formula.



Seven Graceful 75 Foot Span Douglas Fir Hyperbolic-Paraboloids form a 24,000 Square Foot Pavilion.